Decidability Results for the Coverability in Petri Nets with Discrete Parameters

partially presented in Petri Nets 2015
Bruxelles

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October 15, 2015

Why Introducing Parameters ?

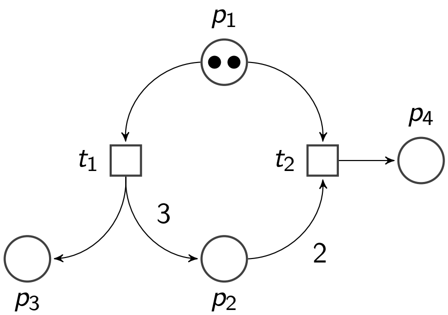
- modeling arbitrary large amount of processes
- modeling unspecified aspect of the environment
- provide a higher level of abstraction

State of the Art

Literature is a bit fragmented: parametric, parameterised or parameterized Petri nets are introduced by diverse authors...

- Parameters are often used to handle dynamic changes in a system (SMPN, ACPN, Strat.PN)
- ω-PN for parametric concurrent systems with dynamic threads creation
- Parameterized Petri nets : parameterization of the structure itself
- Petri Nets where the initial marking of the PN can be parameterised

Classic Model ? a marked Petri Net (PPN)



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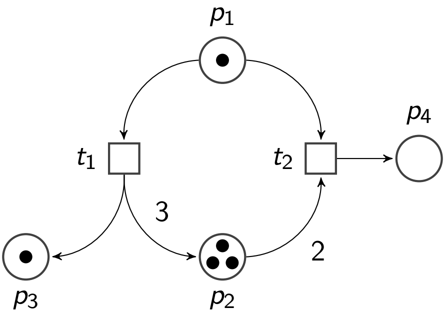

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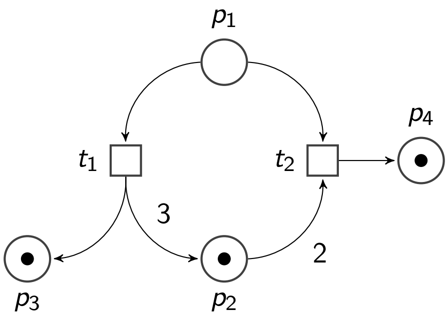
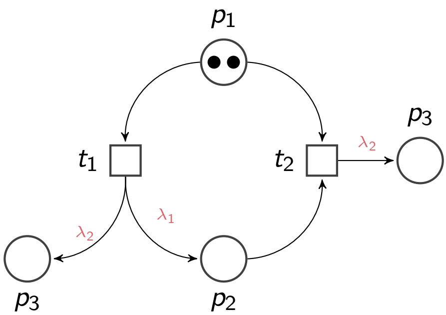


Generalization toward Parametric marked Petri Net (PPN)



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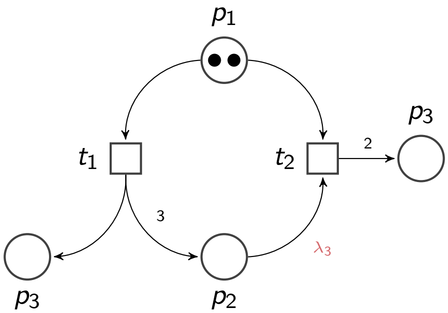

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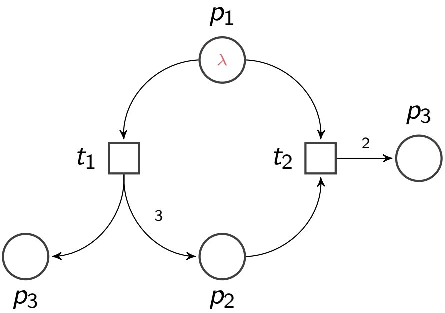

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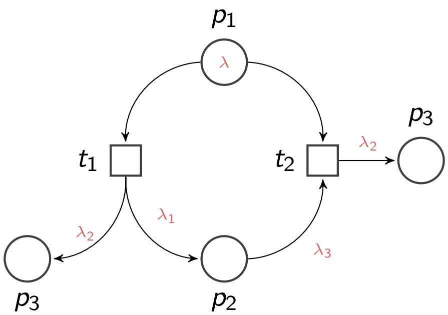
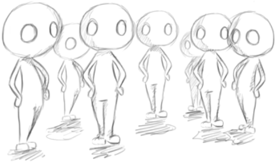
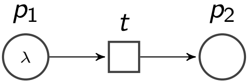
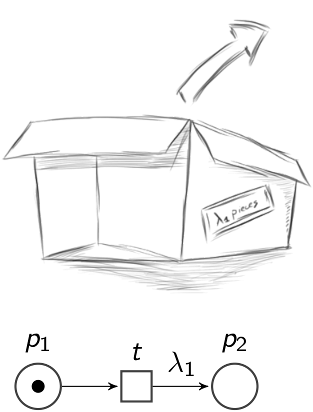


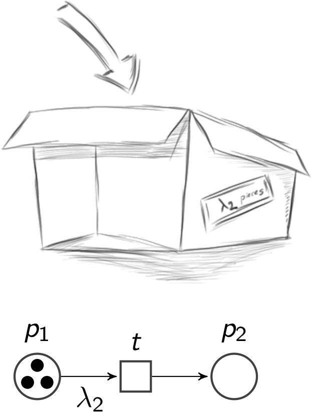
Some Concrete Intuitions (Markings)



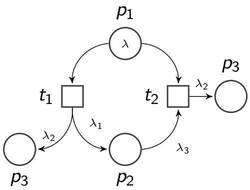


Some Concrete Intuitions (Arcs)

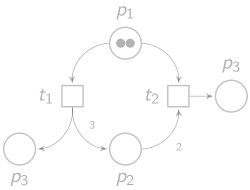




Instantiation



$$\begin{aligned} \nu_1(\lambda) &= 2 \\ \nu_1(\lambda_1) &= 3 \\ \nu_1(\lambda_2) &= 1 \\ \nu_1(\lambda_3) &= 2 \end{aligned}$$



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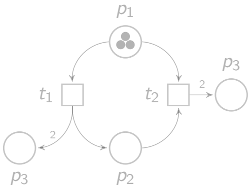

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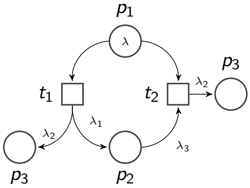

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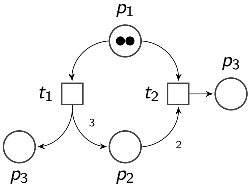

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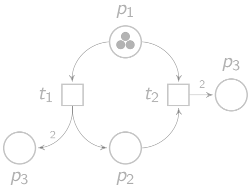

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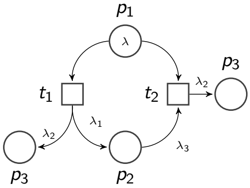

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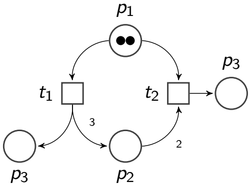

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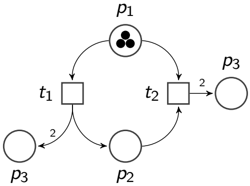


Some Definitions

Definition

Parametric Transitions of $\mathcal{SP}$, $T_{par} \subseteq T$ = set of transitions with "at least one parameter on an input or output"

Definition (Parametric Support)

Given a sequence w, $\Theta(w)$, is the set of parametric transitions involved in w.

$$\Theta(w) = \{t \in T_{par} \text{ s.t. } |w|_t \geq 1\}$$

Reminders...

Let $\mathcal{S} = (\mathcal{N}, m_0) = (P, T, Pre, Post, m_0)$ and m a marking of $\mathcal{S}$

Definition (Reachability)

$\mathcal{S}$ reaches m ($m \in RS(\mathcal{S})$) if there exists a firable sequence of transitions from m_0 to m.

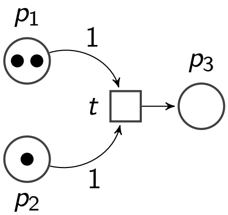
$$\exists w \in T^* \text{ s.t. } m_0 \xrightarrow{w} m \quad (1)$$

Definition (Coverability)

$\mathcal{S}$ covers m if there exists a reachable marking m' of $\mathcal{S}$ such that m' is greater or equal to m i.e.

$$\exists m' \in \mathcal{RS}(\mathcal{S}) \text{s.t. } \forall p \in P, m'(p) \geq m(p) \quad (2)$$

Some Examples



$$RS = \{(2,1,0),(1,0,1)\}$$

$$CS = \{m | m \leq (2,1,0) \vee m \leq (1,0,1)\}$$

Parametric Properties

Given a class of problem $\mathcal{P}$ (coverability, reachability,...), $\mathcal{SP}$ a PPN and ϕ is an instance of $\mathcal{P}$

Definition ($\mathcal{P}$-Existence problem)

($\mathscr{E}$-$\mathcal{P}$): Is there a valuation $\nu \in \mathbb{N}^{Par}$ s.t. $\nu(\mathcal{SP})$ satisfies ϕ ?

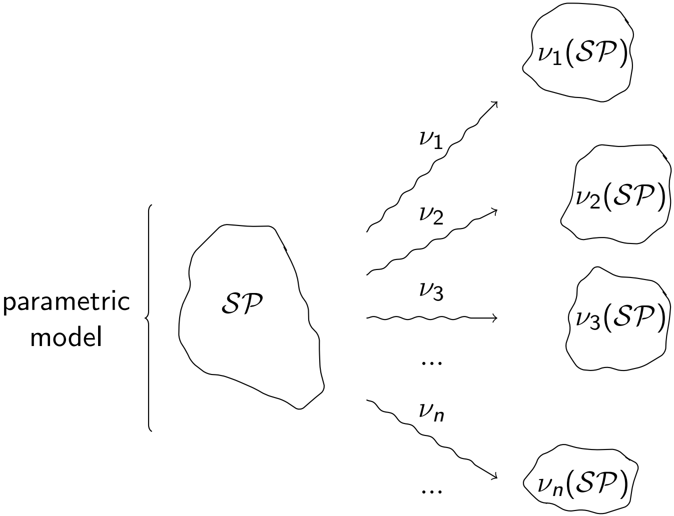
Definition ($\mathcal{P}$-Universality problem)

($\mathscr{U}$-$\mathcal{P}$): Does $\nu(\mathcal{SP})$ satisfies ϕ for each $\nu \in \mathbb{N}^{Par}$?

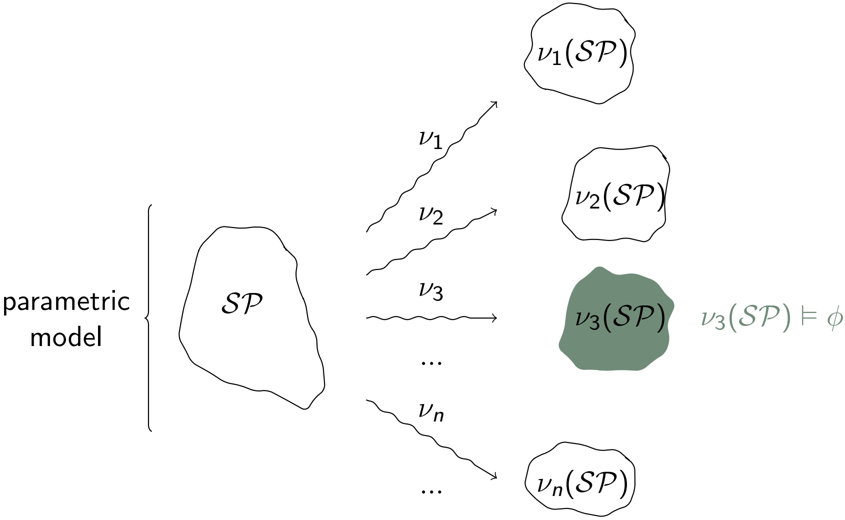
Definition ($\mathcal{P}$-Synthesis problem)

($\mathscr{S}$-$\mathcal{P}$): Give all the valuation ν, s.t. $\nu(\mathcal{SP})$ satisfies ϕ.

Existence


parametric model
$\mathcal{SP}$
ν_1
ν_2
ν_3
...
ν_n
...
$\nu_1(\mathcal{SP})$
$\nu_2(\mathcal{SP})$
$\nu_3(\mathcal{SP})$
$\nu_n(\mathcal{SP})$

Existence



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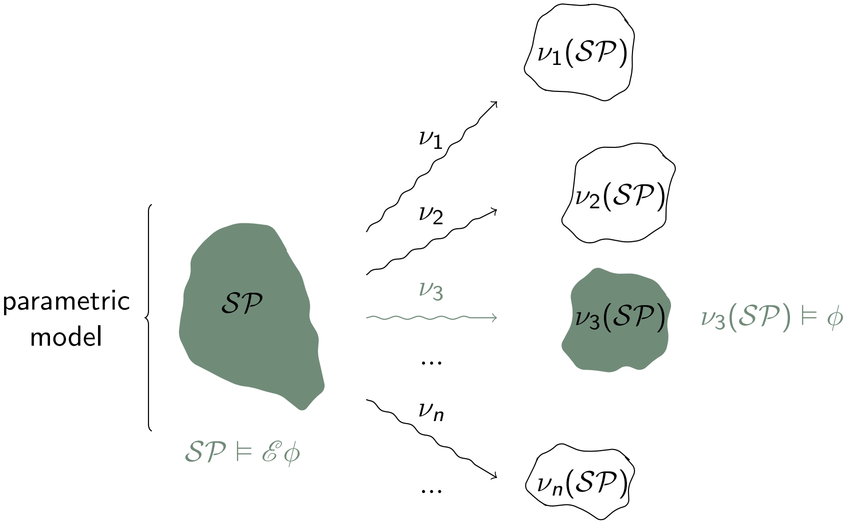
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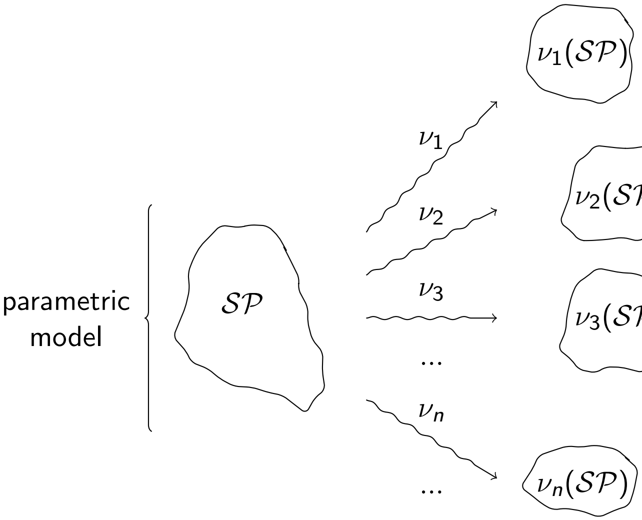
Universality



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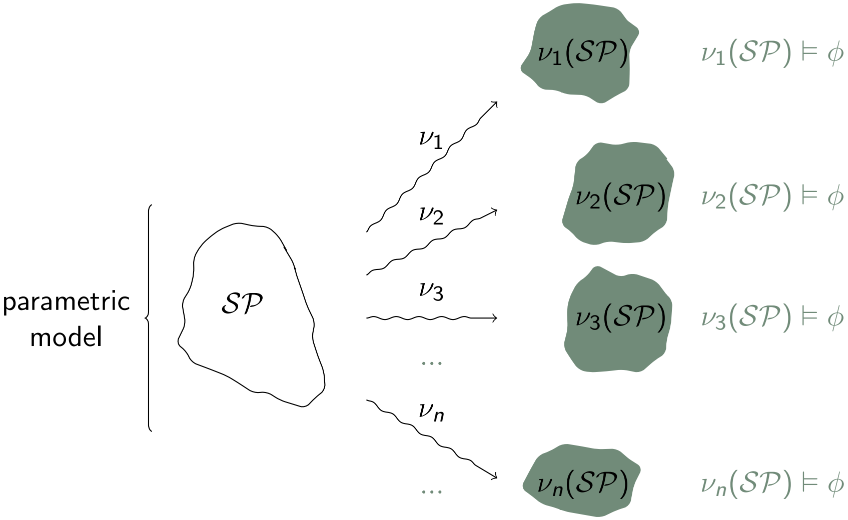
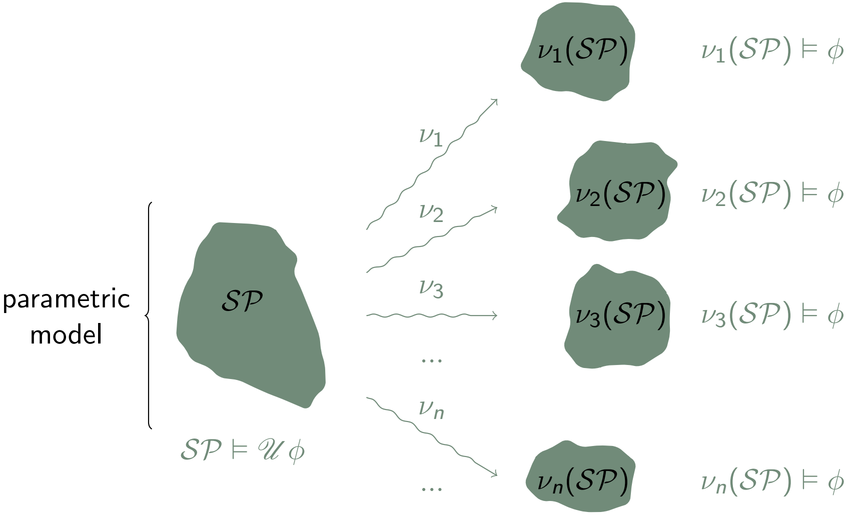


Universality



Results

Theorem (Undecidability of $\mathscr{E}$-cov on PPN)

The $\mathscr{E}$-coverability problem for PPN is undecidable.

Theorem (Undecidability of $\mathscr{U}$-cov on PPN)

The $\mathscr{U}$-coverability problem for PPN is undecidable.

2-Counters Machine

- two counters c_1, c_2,
- states $P = \{p_0, ...p_m\}$, a terminal state labelled *halt*
- finite list of instructions $I_1, ..., I_s$ among the following list:
  - increment a counter
  - decrement a counter
  - check if a counter equals zero

Counters are assumed non negative.

Example of 2-Counters Machine

$$p_1.\ C_0 := C_0 + 1; goto\ p_2;$$

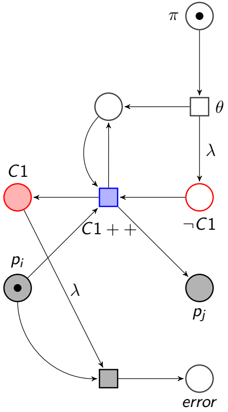
$$p_2.\ C_1 := C_1 + 1; goto\ p_1;$$

instructions sequence:
$(p_1, C_0 = 0, C_1 = 0)$
$\rightarrow (p_2, C_0 = 1, C_1 = 0)$
$\rightarrow (p_1, C_0 = 1, C_1 = 1)$
$\rightarrow (p_2, C_0 = 2, C_1 = 1)$
$\rightarrow ...$

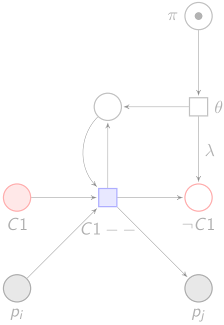
Simulation of a 2-Counters Machine

- $\mathscr{E}$-cov can be reduced to halting problem (whether state *halt* is reachable)
- $\mathscr{U}$-cov can be reduced to counters boundedness problem (whether the counters values stay in a finite set)
- halting problem and counters boundedness problem are undecidable

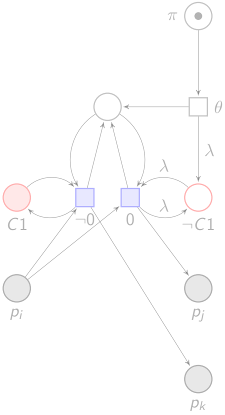
Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$



incrementation
of a counter

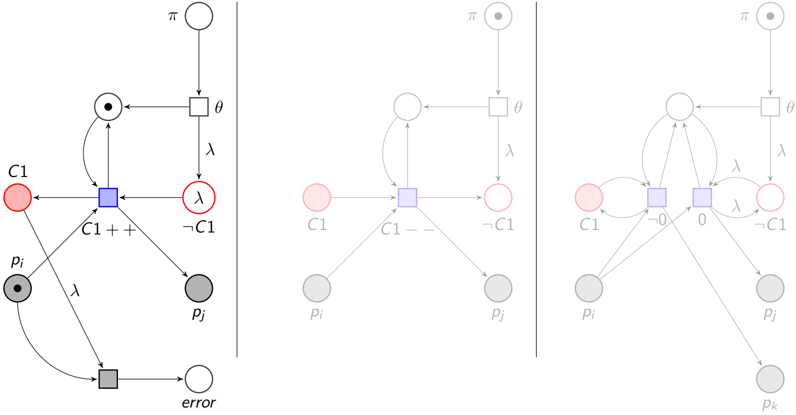


decrementation
of a counter



zero test of
a counter

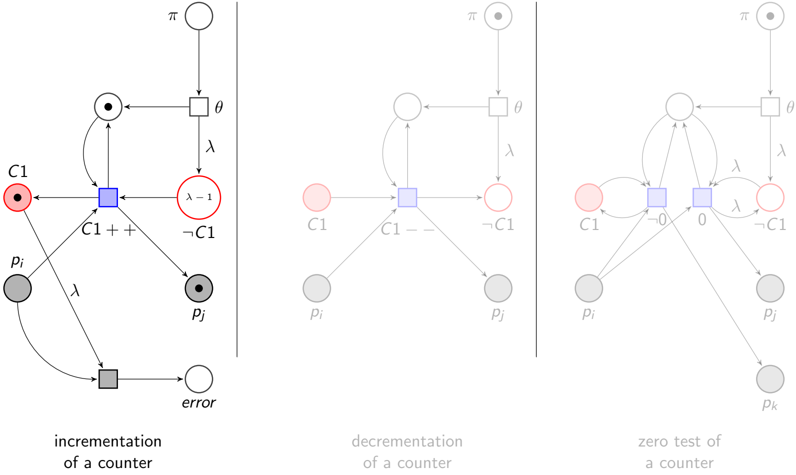
Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$



incrementation of a counter

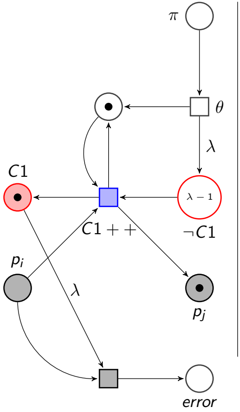
decrementation of a counter

zero test of a counter



π
θ
λ
C1
λ − 1
C1 + +
¬C1
p_i
λ
p_j
error
incrementation
of a counter
π
θ
λ
C1
C1 − −
¬C1
p_i
p_j
decrementation
of a counter
π
θ
λ
λ
λ
C1
¬0
0
¬C1
p_i
p_j
p_k
zero test of
a counter

Simulation of Instructions: $m(C1) + m(\neg C1) = \lambda$



incrementation
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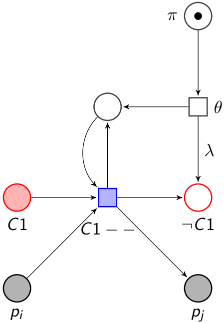

decrementation
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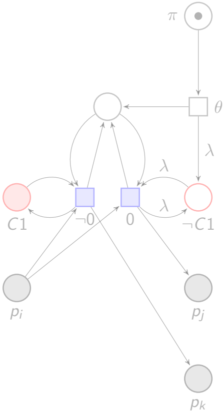

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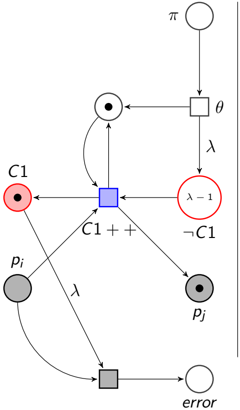

incrementation
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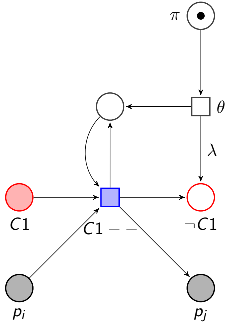

decrementation
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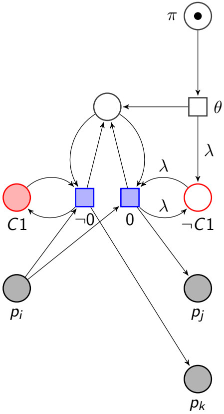
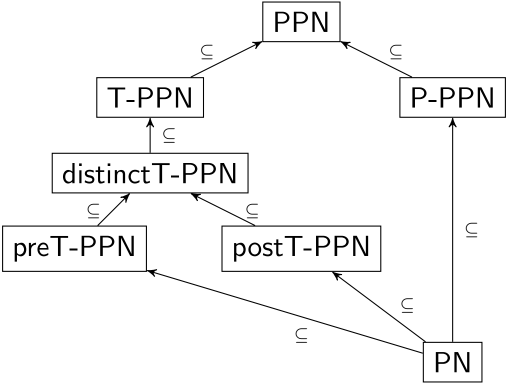
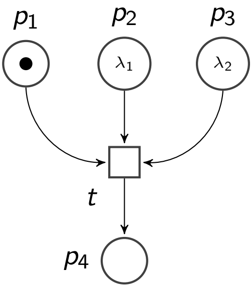


zero test of
a counter

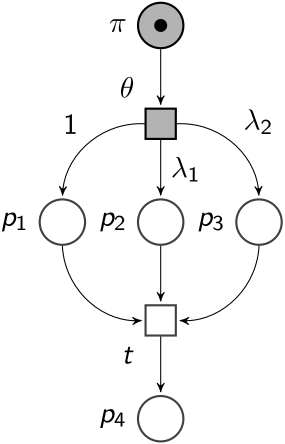
- $\mathcal{M}$ halts iff there exists a valuation ν such that $\nu(\mathcal{SP}_{\mathcal{M}})$ covers the corresponding p_{halt} place.
- the counters are unbounded along the instructions sequence of $\mathcal{M}$ iff for each valuation ν, $\nu(\mathcal{SP}_{\mathcal{M}})$ covers the *error state*.

Hierarchy of Parametric Petri Nets

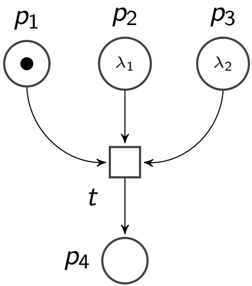




replacement of
the **P parameters**
by **postT**
parameters



From Markings to Output Weights



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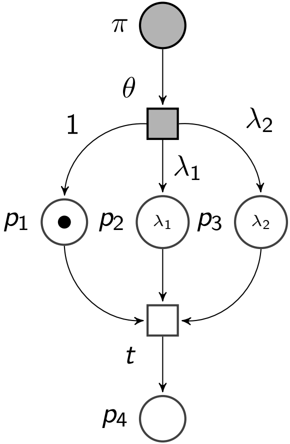
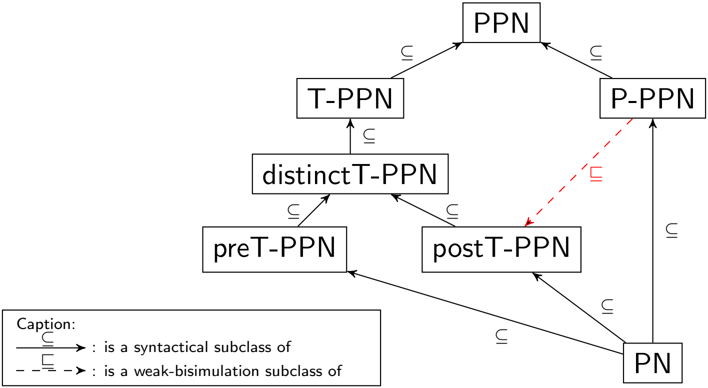
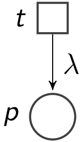


Expand the Hierarchy (1)



From Output Weights to Markings



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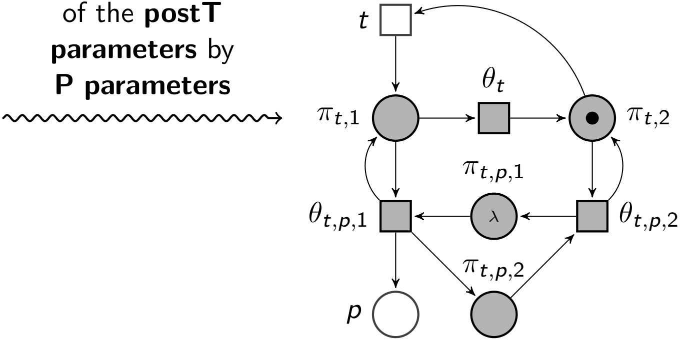

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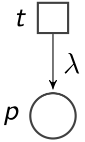

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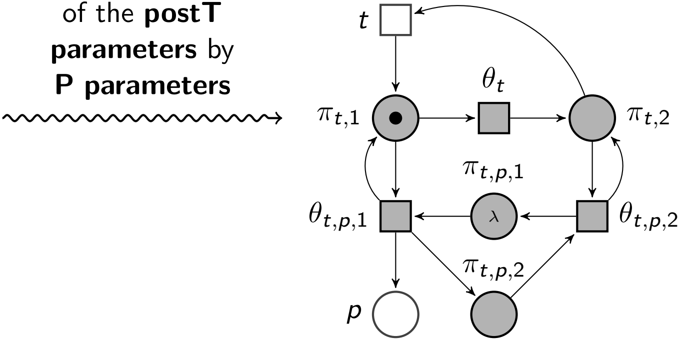

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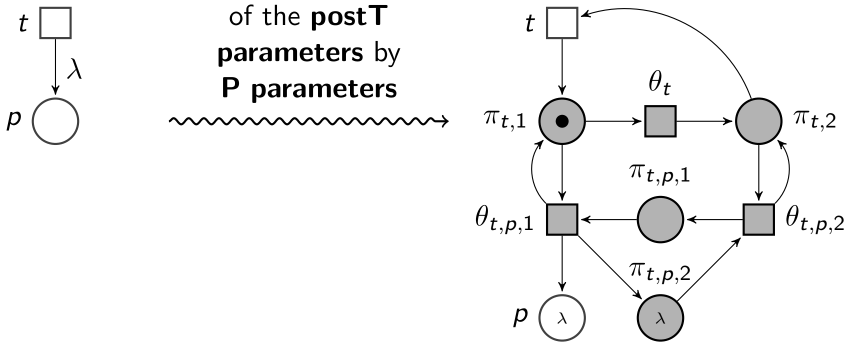

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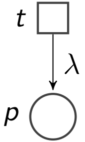

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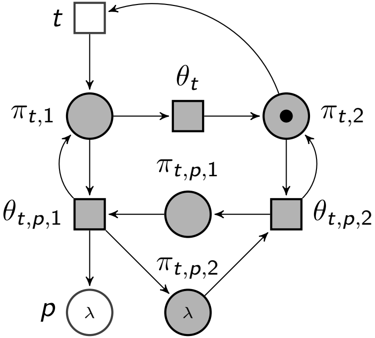

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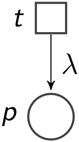

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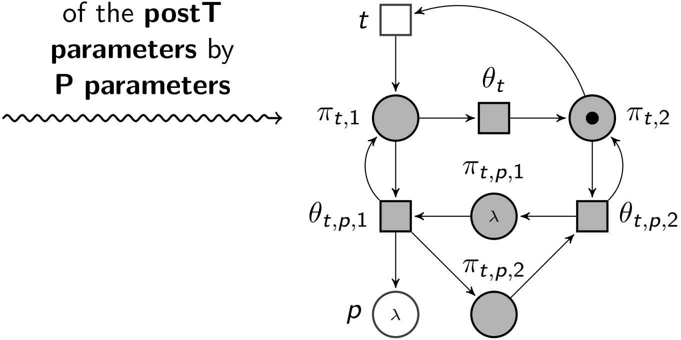
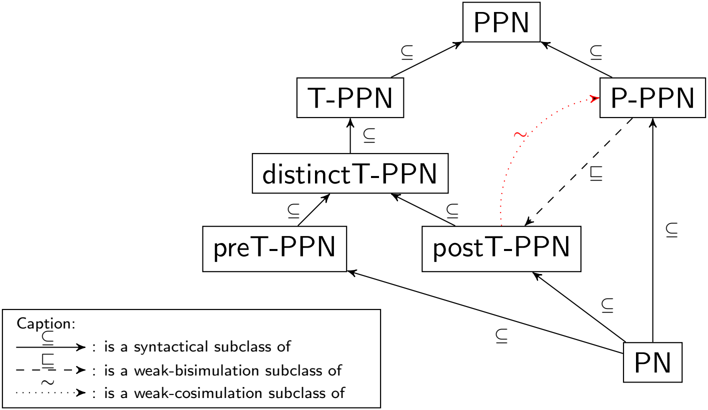
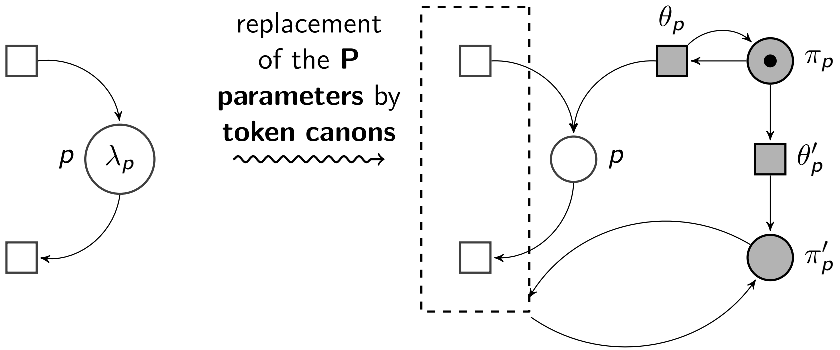


Expand the Hierarchy (2)



From Parametric Markings to Classic Petri Nets



Monotony in preT-PPN

Let $\mathcal{SP} = (P, T, Pre, Post, Par)$ be a preT-PPN.

Lemma

Let $w \in T^*$ be a transitions sequence such that $m_0 \xrightarrow{w}_\nu m$. For any valuation $\nu' \leq \nu$, $m_0 \xrightarrow{w}_{\nu'} m'$ s.t. $m' \geq m$

We obtain that for any $\nu_i < \nu_j$,

$$\mathcal{CS}(\nu_j(\mathcal{SP})) \subseteq \mathcal{CS}(\nu_i(\mathcal{SP})) \tag{3}$$

Thus :

- Set of markings coverable in at least one instance = $\mathcal{CS}(\mathbf{0}(\mathcal{SP}))$
- Set of markings coverable in any instance = $\mathcal{CS}(\omega(\mathcal{SP}))$.

Monotony in postT-PPN

Let $\mathcal{SP} = (P, T, Pre, Post, Par)$ be a postT-PPN.

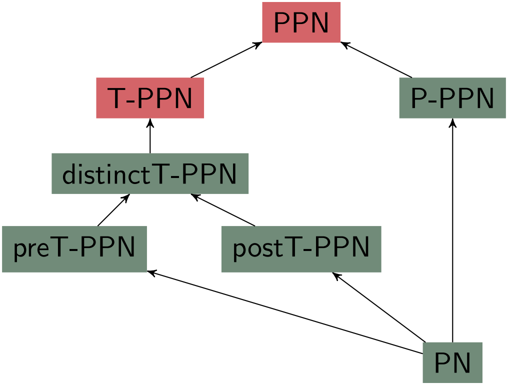
Lemma

Let $w \in T^*$ be a transitions sequence such that $m_0 \xrightarrow{w}_{\nu} m$. For any valuation ν' such that $\nu \leq \nu'$, $m_0 \xrightarrow{w}_{\nu'} m'$ s.t. $m' \geq m$

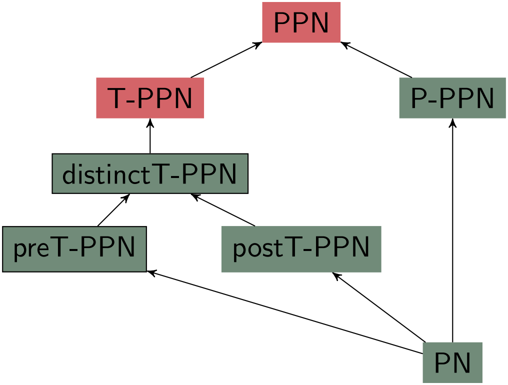
Thus:

- Set of markings coverable in at least one instance $=$ $\mathcal{CS}(\omega(\mathcal{SP}))$
- Set of markings coverable in any instance $= \mathcal{CS}(\mathbf{0}(\mathcal{SP}))$.

Decidability of the $\mathscr{E}$-Coverability



Decidability of the $\mathscr{U}$-Coverability



Ideas of the Proofs

- Monotony in postT-PPN gives $\mathscr{U} - cov$
- Monotony in preT-PPN gives $\mathscr{E} - cov$
- Translation from P-PPN in PN gives $\mathscr{E} - cov$
- Weak bisimilarity between postT-PPN and P-PPN complete the study of postT & P-PPN
- Adaptation of Karp and Miller Algorithm provides $\mathscr{U} - cov$ in preT-PPN
- In distinctT-PPN : partition of parameters Par_{Pre} and Par_{Post} with previous results

Detail of $\mathscr{U} - cov$ in preT-PPN

Marking universally coverable ⇒ two main cases:

1. cover this marking without using any parametric transition
2. otherwise, there exists a run which uses parametric transition that could be fired in any instance of the preT-PPN

(2) ⇒ the firing condition of a parametric arcs involved can be covered for any valuation ⇒ acceleration of markings reached creates ω's in the input places of the parametric arcs involved.

Precise the Study of Coverbality

	$\mathscr{U}$-Coverability	$\mathscr{E}$-Coverability
Subclasse	Complexity	Complexity
preT-PPN	EXPSPACE-h	EXPSPACE-c
postT-PPN	EXPSPACE-c	EXPSPACE-c
PPN	Undecidable	Undecidable
distinctT-PPN	EXPSPACE-h	EXPSPACE-c
P-PPN	EXPSPACE-c	EXPSPACE-c

Table 1: Decidability results for parametric coverability and reachability

Toward Synthesis

- A backward algorithm using upward closed sets.
- Algorithm : Symbolic Exploration of the state space by constraints and projection on the parameters with Parma Polyhedra Lib. (C++)

Contributions

- Parameterised models with higher level of abstraction
- Parameters on input arcs, output arcs and markings
- Study of decidability of parameterised versions of well-known properties

Future Work

- ~~Prove that the Universal coverability is decidable on Petri Nets with parameters on input arcs~~ Done
- Extend this study to reachability *($\mathscr{E}$-reachability is decidable for Nets with parameters on markings)*
- Study of the Synthesis Problem *(using symbolic constraints)*

Thank you for your attention

any remarks ?
any questions ?