Counterfactual Explanations Under Learning From Interpretation Transition

Tony Ribeiro[1,3,4], Maxime Folschette[2], Morgan Magnin[1], Katsumi Inoue[3], Kotaro Okazaki[4], Kuo-Yen Lo[4], Tuan Nguyen[5], Jérémie Poschmann[6], Antoine Roquilly[6,7]

1. Nantes Université, École Centrale Nantes, CNRS, LS2N, UMR 6004, F-44000 Nantes, France
2. Univ. Lille, CNRS, Centrale Lille, UMR 9189 CRIStAL, F-59000 Lille, France.
3. National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan.
4. Steelous Protocol, 8-20-32, Ginza, Chuo-ku, Tokyo 104-0061, Japan.
5. Hanoi University of Science and Technology, No. 1 Dai Co Viet, Hai Ba Trung, Ha Noi, Vietnam.
6. Inserm, Nantes Université, CHU Nantes, Center for Research in Transplantation and Translational Immunology, UMR 1064, ITUN2, F-44000, Nantes, France.
7. Nantes Université, CHU Nantes, INSERM, Anesthesie Réanimation, F-44000 Nantes, France.

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Outline

Outline

1. Motivations
2. LFIT Overview
3. Counterfactual under LFIT
   - CELOS Algorithm
4. Conclusions

HOMI-LUNG: A strong consortium to fill the gap.
Pneumonia is a heart matter too (and vice-versa).

HOMI-LUNG

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Motivations: machine-produced Decision Making

Algorithms used for decision-making impact human lives in many domains.

- Credit lending
- Talent sourcing
- Medical treatment

Rising interests regarding fairness, accountability and transparency.

Motivations: eXplainable Artificial Intelligence (XAI)

Explanations for machine-produced decisions has several motivations.

Organization benefit

- Useful to check for bias and unsure fairness.
- Can help the models developers identify performance issues.
- Help adhere to laws for machine-produced decisions, e.g., GDPR

User benefits:

- Personalized feedback highlights key factors influencing outcome.
- Empowering actionable insights to lead to favorable outcomes.
- Various explanations serve transparency and trustworthiness.

Counterfactual Explanations

Counterfactuals involve considering alternatives that affect decisions.

Example

Bank denied a loan application because: "credit score is insufficient".

- It does not give insight to the customer to change the outcome.

Counterfactual explanations:

- "Annual income is 30,000$, changing to 45,000$ makes loan ok"

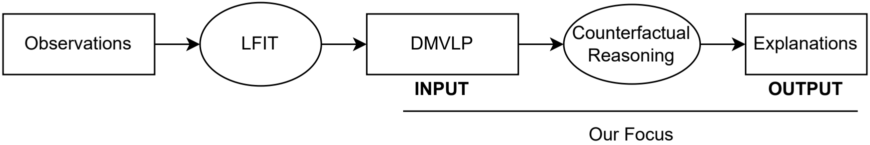
In this example we get two kind of insight from counterfactuals:

- Decision causes: Income below threshold led to denial.
- Potential solution: increase income by 15,000$ for approval.

It allows to comprehend actions taken and avoid similar issues in future.

Counterfactual reasoning from LFIT

Goal: produce counterfactual explanations from LFIT output.



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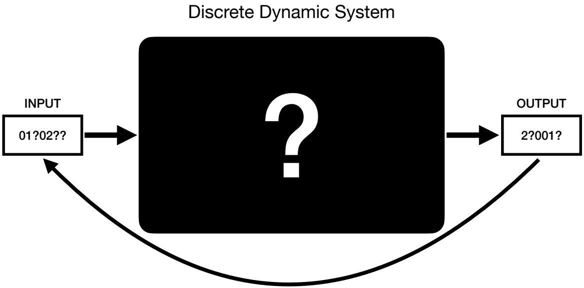
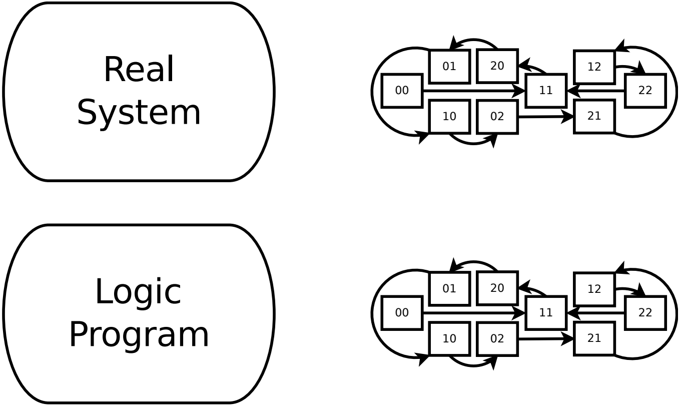
Learning From Interpretation Transitions



- **Idea:**
  Learn black-box **internal mechanics** from its input/output states.
- **Discrete System:**
  Input/output are vectors containing discrete and unknowns values.
- **Dynamic System:**
  System output become the next input (require vectors of same size).

Learning From Interpretation Transitions

Goal: produce a logic program with the same behavior as the system observed.



Logic Rule

$$\underbrace{\mathrm{v}_0^{val_0}}_{\substack{\textit{head}\\ \text{target atom}}} \leftarrow \underbrace{\mathrm{v}_1^{val_1} \wedge \mathrm{v}_2^{val_2} \wedge \ldots \wedge \mathrm{v}_n^{val_n}}_{\substack{\textit{body}\\ \text{feature atoms}}}.$$

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- v_0, v_1, v_2, ..., v_n: variables a_t, a_{t-1}, b_t, b_{t-1}, z_t, z_{t-1}
  - Variables are split into feature ($\mathcal{F}$) and target ($\mathcal{T}$) variables
  - $\mathrm{v}_0 \in \mathcal{T}$ a_t, b_t, z_t
  - $\mathrm{v}_1, \mathrm{v}_2, \dots, \mathrm{v}_n \in \mathcal{F}$ a_{t-1}, b_{t-1}, z_{t-1}
  - Implicit time step: t in the *head* and $t-1$ in the *body*

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  - $\mathrm{v}_1, \mathrm{v}_2, \ldots, \mathrm{v}_n \in \mathcal{F}$ $a_{t-1},\ b_{t-1},\ z_{t-1}$
  - Implicit time step: t in the *head* and $t-1$ in the *body*
- val_0, val_1, val_2, ..., val_n: values 0, 1, 2, ...
  - $val_i \in \mathrm{dom}(\mathrm{v}_i)$

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  - Implicit time step: t in the *head* and $t-1$ in the *body*
- $val_0, val_1, val_2, \ldots, val_n$: values $\quad 0, 1, 2, \ldots$
  - $val_i \in \mathrm{dom}(\mathrm{v}_i)$
- All atoms in the *body* are in conjunction
- $\leftarrow$ is the (reverse) implication

Interpretation of a Logic Rule

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Interpretation: When *body* is true, *head* is a potential outcome

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Examples:

$$a_t^1 \leftarrow a_{t-1}^2 \wedge b_{t-1}^0 \wedge z_{t-1}^1.$$
$$b_t^1 \leftarrow z_{t-1}^1.$$
$$z_t^0 \leftarrow \top.$$

Interpretation of a Logic Rule

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$$\text{Examples:} \quad \left.\begin{array}{l} a_t^1 \leftarrow a_{t-1}^2 \wedge b_{t-1}^0 \wedge z_{t-1}^1. \\ b_t^1 \leftarrow z_{t-1}^1. \\ z_t^0 \leftarrow \top. \end{array}\right\} \text{ all } \textbf{match } \langle a_{t-1}^2, b_{t-1}^0, z_{t-1}^1 \rangle$$

A rule R **matches** a state s iff $body \subseteq s$

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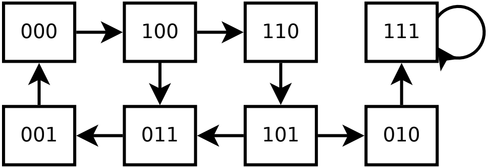
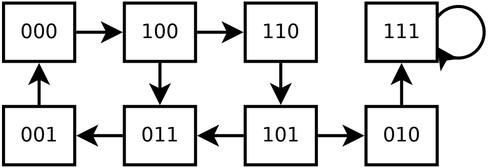
Semantics = From this information, what are the next possible state(s)?
(Similar to discrete networks)

Example: LFIT with complete observations



Features: $\{a_{t-1}, b_{t-1}, c_{t-1}\}$, Targets: $\{a_t, b_t, c_t\}$, Domains: $\{0, 1\}$

Example: LFIT with complete observations



Features: $\{a_{t-1}, b_{t-1}, c_{t-1}\}$, Targets: $\{a_t, b_t, c_t\}$, Domains: $\{0, 1\}$

$a_t^1 \leftarrow c_{t-1}^0$
$a_t^1 \leftarrow a_{t-1}^1, b_{t-1}^1$

$a_t^0 \leftarrow a_{t-1}^0, c_{t-1}^1$
$a_t^0 \leftarrow a_{t-1}^1, b_{t-1}^0$
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$b_t^1 \leftarrow a_{t-1}^1, b_{t-1}^0$
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$c_t^1 \leftarrow a_{t-1}^1$
$c_t^1 \leftarrow b_{t-1}^1$

$c_t^0 \leftarrow b_{t-1}^0$

Outline

Problem Definition

Definition (Counterfactual Explanation Problem)

Let P be a $\mathcal{DM}\mathrm{VLP}$, $s \in \mathcal{S}^{\mathcal{F}}$ be a feature state, $\mathrm{v} \in \mathcal{T}$ be a target variable. A counterfactual explanation problem is a tuple $CP := (P, s, \mathrm{v}, Val_{out}, Val_{in})$ where $Val_{out} \subset \mathrm{dom}(\mathrm{v})$ and $Val_{in} \subset \mathrm{dom}(\mathrm{v})$ such that $Val_{out} \cap Val_{in} = \emptyset$. A solution to CP is a set of atoms $X = s' \setminus s$ for some feature state s' such that:

- no rule R of P with $head(R) = \mathrm{v}^{val}, val \in Val_{out}$ matches s' and
- there is a rule R' of P with $head(R') = \mathrm{v}^{val'}, val' \in Val_{in}$ that matches s'.

A solution to CP is minimal if no subset of the solution is also a solution.

Counterfactual Explanation Example

Example

Consider the following $\mathcal{DM}\mathrm{VLP}$ P such that $\mathcal{F} = \{a, b, c\}, \mathcal{T} = \{y\}, \mathrm{dom}(a) = \mathrm{dom}(b) = \{0, 1\}, \mathrm{dom}(c) = \mathrm{dom}(y) = \{0, 1, 2\}$:

$$
\begin{array}{lll}
y^0 \leftarrow a^0 & y^1 \leftarrow b^1 & y^2 \leftarrow a^1 \wedge b^1 \wedge c^2 \\
y^0 \leftarrow b^0 & y^1 \leftarrow a^0 \wedge c^1 & \\
y^0 \leftarrow c^0 & y^1 \leftarrow c^2 &
\end{array}
$$

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Let $CP := (P, s, y, \{y^1\}, \{y^0, y^2\})$ where $s = \{a^0, b^1, c^1\}$.

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$y^0 \leftarrow a^0$	$y^1 \leftarrow b^1$	$y^2 \leftarrow a^1 \wedge b^1 \wedge c^2$
$y^0 \leftarrow b^0$	$y^1 \leftarrow a^0 \wedge c^1$	
$y^0 \leftarrow c^0$	$y^1 \leftarrow c^2$	

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y^0 \leftarrow c^0 & y^1 \leftarrow c^2 &
\end{array}
$$

Let $CP := (P, s, y, \{y^1\}, \{y^0, y^2\})$ where $s = \{a^0, b^1, c^1\}$.

- The minimal solutions for y^0 are $\{\{a^1, b^0\}, \{b^0, c^0\}\}$.

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Let $CP := (P, s, y, \{y^1\}, \{y^0, y^2\})$ where $s = \{a^0, b^1, c^1\}$.

- The minimal solutions for y^0 are $\{\{a^1, b^0\}, \{b^0, c^0\}\}$.
- No solution for y^2.

Naïve enumeration approach

- **INPUT:** $CP := (P, s, \mathrm{v}, Val_{out}, Val_{in})$.
- $out\_rules := \{R \in P \mid head(R) = \mathrm{v}^{val}, val \in Val_{out}\}$
- For each val of $Val_{in}, in\_rules_{val} := \{R \in P \mid head(R) = \mathrm{v}^{val}\}$
- For each val of $Val_{in}, solutions_{val} := \emptyset$
- For each complete state $s' \in \mathcal{S}^{\mathcal{F}}$:
  - If a rule of out_rules matches s': // Discard invalid state
    - ★ Continue
  - For each val of Val_{in}: // Extract valid changes to get v^{val}
    - ★ If a rule of in_rules_{val} matches s':
      $solutions_{val} := solutions_{val} \cup \{\{x \in s' \mid x \notin s\}\}$
- For each val of Val_{in}: // Keep only minimal sets
  - $solutions_{val} := \{x \in solutions_{val} \mid \nexists x' \in solutions_{val}, x' \subset x\}$
- **OUTPUT:** $\{solutions_{val} \mid val \in Val_{in}\}$

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Cross-matching

Definition (Rule cross-matching)

Let R, R' be two $\mathcal{M}\mathrm{VL}$ rules. The rules cross-match, if there exists a feature state $s \in \mathcal{S}^{\mathcal{F}}$ such that both R and R' match s.

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Example

Let $\mathcal{A} = \{a, b, c, y\}$, domain of a, b is $\{0, 1\}$, domain of c, y is $\{0, 1, 2\}$:

- $y^1 \leftarrow a^0$, $y^1 \leftarrow b^1$ cross-match $\{a^0, b^1, c^0\}, \{a^0, b^1, c^1\}, \{a^0, b^1, c^2\}$.
- $y^1 \leftarrow b^1 \wedge c^0$ and $y^1 \leftarrow b^1$ cross-match $\{a^0, b^1, c^0\}, \{a^1, b^1, c^0\}$.
- $y^1 \leftarrow a^0 \wedge b^0$ and $y^1 \leftarrow b^1$ do not cross-match.

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- $y^1 \leftarrow b^1 \wedge c^0$ and $y^1 \leftarrow b^1$ cross-match $\{a^0, b^1, c^0\}, \{a^1, b^1, c^0\}$.
- $y^1 \leftarrow a^0 \wedge b^0$ and $y^1 \leftarrow b^1$ do not cross-match.

Proposition

Let R, R' be two $\mathcal{M}\mathrm{VL}$ rules, they cross-match if and only if: for any v^{val} of $body(R)$, if there exists $\mathrm{v}^{val'} \in body(R')$, it implies that $val = val'$.

Anti-cross matching least specialization

Definition (Anti cross-matching least specialization)

Let R, R' be two $\mathcal{M}\mathrm{VL}$ rules that cross-match. The anti cross-matching least specialization of R by R' according to $\mathcal{A}$ is:

$$ACML_{\mathrm{spe}}(R, R', \mathcal{A}) := \{head(R) \leftarrow body(R) \cup \{\mathrm{v}^{val}\} \mid \mathrm{v}^{val'} \in body(R'), \mathrm{v}^{val} \in \mathcal{A}, val \neq val'\}.$$

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Proposition

Let R, R' be two $\mathcal{M}\mathrm{VL}$ rules. It holds that:

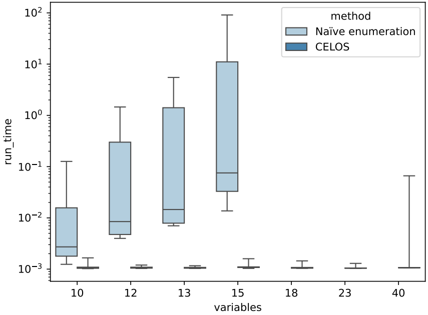
1. *For any R'' of $ACML_{\mathrm{spe}}(R, R', \mathcal{A})$, R'' and R' do not cross-match.*
2. *The feature states matched by $ACML_{\mathrm{spe}}(R, R', \mathcal{A})$ are all the states matched by R but not by R':*

$$\{s \in \mathcal{S}^{\mathcal{F}} \mid R'' \in ACML_{\mathrm{spe}}(R, R', \mathcal{A}), R'' \sqcap s\} = \{s \in \mathcal{S}^{\mathcal{F}} \mid R \sqcap s, R' \not\sqcap s\}.$$

CELOS

- **INPUT:** $CP := (P, s, \mathrm{v}, Val_{out}, Val_{in})$.
- $out\_rules := \{R \in P \mid head(R) = \mathrm{v}^{val}, val \in Val_{out}\}$
- For each val of Val_{in}, $in\_rules_{val} := \{R \in P \mid head(R) = \mathrm{v}^{val}\}$
- For each val of Val_{in}, $solutions_{val} := \emptyset$
- Initialize $P_{in} := \{\_ \leftarrow \emptyset\}$ // Rule head does not matter, only body is used
- For each rule R' of out_rules: // 1) Search necessary changes to avoid all Val_{out}
  - Extract and remove the rules of P_{in} that cross-match R':
    $M := \{R \in P_{in} \mid \forall \mathrm{v}^{val} \in body(R), \mathrm{v}^{val'} \in R' \implies val = val'\}$
  - $P_{in} := P_{in} \setminus M$
  - $LS := \emptyset$
  - For each rule R of M:
    - Compute its rule least specialization $P'_{in} = ACML_{\mathrm{spe}}(R, R', \mathcal{A})$
    - Remove rules in P'_{in} dominated by a rule in P_{in}
    - $LS := LS \cup P'_{in}$
  - Add all remaining rules of LS to P_{in}: $P_{in} := P_{in} \cup LS$
- For each val of Val_{in}: // 2) Compute necessary changes to produce a Val_{in}
  - For each R of in_rules_{val}, for each R' of in_rules_{val}:
    - If R, R' cross-match: // Combine and extract changes with state s
      $candidate := \{x \in (body(R) \cup body(R')) \mid x \notin s\}$
      $solutions_{val} := solutions_{val} \cup \{candidate\}$
  - $solutions_{val} := \{S \in solutions_{val} \mid \nexists S' \in solutions_{val}, S' \subset S\}$
- **OUTPUT:** $\{solutions_{val} \mid val \in Val_{in}\}$

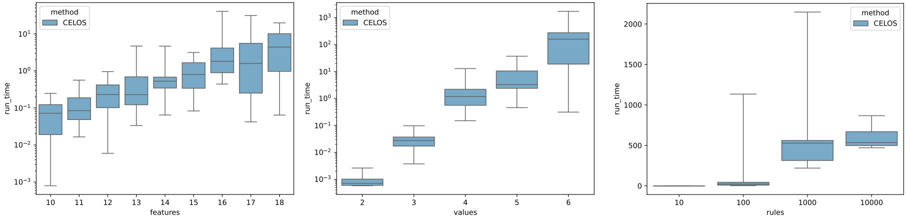
Evaluation



Runtime (in seconds, log scale) comparison of **CELOS** and **Naïve enumeration** on random counterfactual explanation problems.

- Boolean networks from Biological literature with 10 to 40 variables.
- 100 runs per target variable value, with a timeout of $1,000$ seconds.

Evaluation



Runtime (in seconds, log scale) of **CELOS** when applied to random counterfactual explanation problems from synthetic $\mathcal{DM}\mathrm{VLP}$ with a single target variable. The experiments used 10 runs for each problem setting.

- (1) Varying the number of variables from 10 to 18.
- (2) Increasing the domain size from 2 to 6.
- (3) Scaling the number of rules per target from 10 to $10,000$.

Outline

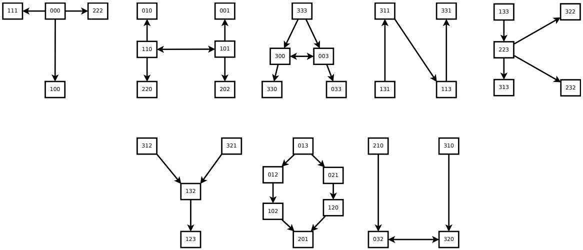
Conclusions

Contributions:

- Formalization of counterfactuals for dynamic multi-valued logic
- CELOS an efficient algorithm that leverage properties of DMVLP rules

Applications:

- Can be use to get more insight about the system learned by LFIT
- Analyze what can change a patient prognostic: decision robustness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