# Asymptotic behavior of a degenerate forest kinematic model with a perturbation

**Lu LI**

Co-Author: Guillaume CANTIN

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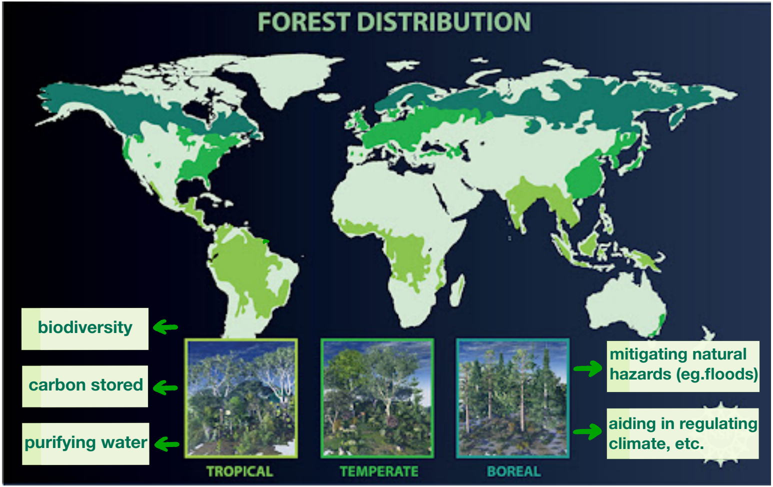
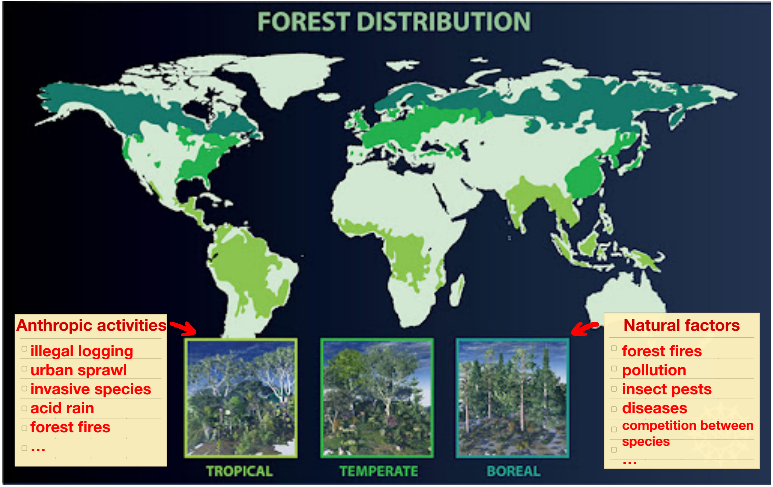
# Summary

# Section 1

- Ecosystem services



- Factors impact on forests



- The forest kinematic model[1][2]:

$$\begin{cases} \frac{\partial u}{\partial t} = \beta\delta w - \gamma(v)u - fu & \text{in } (0, +\infty) \times \Omega, \\ \frac{\partial v}{\partial t} = fu - hv & \text{in } (0, +\infty) \times \Omega, \\ \frac{\partial w}{\partial t} = d\Delta w - \beta w + \alpha v & \text{in } (0, +\infty) \times \Omega, \end{cases} \tag{1}$$

where

$$\gamma(v) = a(v-b)^2 + c.$$

$u$: the density of young age class trees;
$v$: the density of old age class trees;
$w$: the density of seeds in the air.

[1] Yu.A. Kuznetsov, M.Ya. Antonovsky, V. Biktashev and E.A. Aponina, A cross-diffusion model of forest boundary dynamics, Journal of Mathematical Biology, 32(1994)219–232

[2] A. Yagi, Abstract parabolic evolution equations and their applications, Springer Science & Business Media, 2009.

# Section 2

- The degenerate forest kinematic model

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha w - q_\mu(u) & \text{in } (0, +\infty) \times \Omega, \\ \frac{\partial w}{\partial t} = \delta \Delta w - \beta w + \alpha u & \text{in } (0, +\infty) \times \Omega, \end{cases} \tag{2}$$

where

$$q_\mu = q(u) + \mu p(u), \quad q(u) = u[a(b-u)^2 + c].$$

$u$: density of the trees;
$w$: density of the air-borne seeds;
$\mu$: perturbation parameter.

# Main results

- well-posedness results: local/global solutions
- Convergence towards equilibrium (Łojasiewicz-Simon gradient inequality[3]) under restriction: $q_\mu$ is monotone[4]
  $U_\mu(t) \to \bar{U}_\mu$ in $Y$ as $t \to \infty$.
- Robustness of the weak attractors
  - continuity of the flow:

$$\|S_\mu(t)U_0 - S_0(t)U_0\|_Y^2 \leq \frac{\mu^2 M_1^2 |\Omega|}{q_0(\alpha - \varrho)}(e^{2(\alpha-\varrho)t} - 1). \tag{4.8}$$

*Moreover,*

*(i) if* $\alpha - \varrho < 0$, *then* $U_\mu(t) \overset{\mu \to 0^+}{\longrightarrow} U(t)$ *in* $Y$ *uniformly for* $t \in [0, +\infty)$*;*

*(ii) if* $\alpha - \varrho > 0$, *then* $U_\mu(t) \overset{\mu \to 0^+}{\longrightarrow} U(t)$ *in* $Y$ *uniformly in every compact interval* $[0, T]$ *with* $T > 0$.

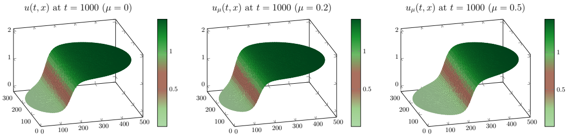
  - continuity of the stationary solutions:
    if $\alpha - \varrho < 0$, then $\bar{U}_\mu \overset{\mu \to 0}{\longrightarrow} \bar{U}$ uniformly in $Y$.
  - robustness of the weak attractors.

[3] S. Iwasaki, Asymptotic convergence of solutions to the forest kinematic model, Nonlinear Analysis: Real World Applications, 62(2021)103382.

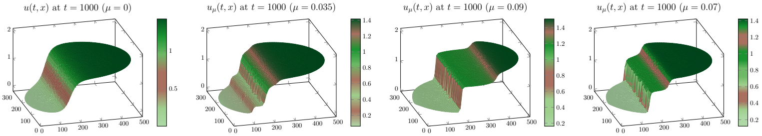
[4] Efendiev, M. and Zelik, S., Global attractor and stabilization for a coupled PDE-ODE system,(2011)1–23, arXiv:1110.1837v1.

# Main results

- Numerical simulations
  - Shift of the ecotone;

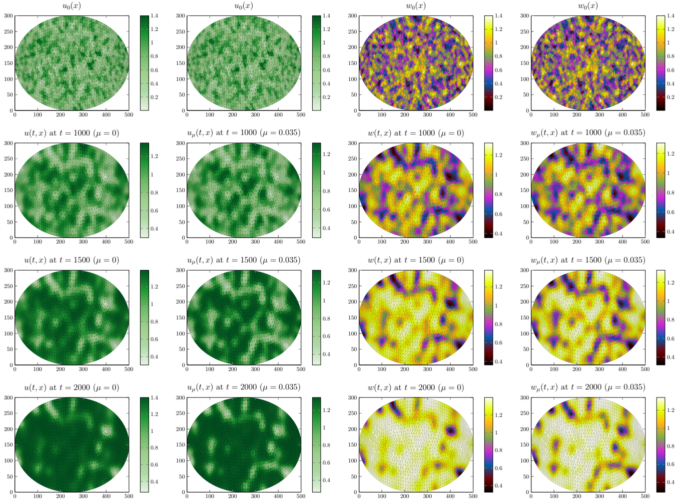



  - Emergence of intermediate ecosystems;




  - Randomly generated initial conditions lead to chaotic patterns.

# Main results



# Section 3

Recall the degenerate forest kinematic model:

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha w - q_\mu(u) & \text{in } (0,+\infty)\times\Omega, \\ \frac{\partial w}{\partial t} = \delta\Delta w - \beta w + \alpha u & \text{in } (0,+\infty)\times\Omega, \end{cases} \tag{3}$$

where

$$q_\mu = q(u) + \mu p(u), \quad q(u) = u[a(u-b)^2 + c].$$

- $c(x)$: the mortality of trees, which depends on the water resource (precipitation, soil moisture, evapotranspiration) with respect to the space. How to deal with the real world spatial data?
- Geographers observed the disappear and emergence of trees in some zones of the forest, and the transform of the forest. How to explain this from mathematical point of view to help ecologists to better understand the forest ecosystem?

# Related work

Computational assessment of Amazon forest plots regrowth capacity under strong spatial variability for simulating logging scenarios

December 3, 2023

Gilles Ardourel[1], Guillaume Cantin[1,2], Benoît Delahaye[1], Géraldine Derroire[3], Beatriz M. Funatsu[4], David Julien[1]

**Abstract**

In this paper, we assess the regrowth capacity of tropical forest plots by developing an original computational procedure based on statistical model checking methods. We calibrate a new mathematical model of forest dynamics with respect to post-logging data, produced in the Amazon basin. Rather than a single set of parameters, our method returns a small cell of parameters, extracted from a huge parameter space, which contains in its close vicinity several relevant sets of parameters that are equally able to reproduce the regrowth dynamics of distinct tropical forest plots, as well as the high level of biological variability identified between these forest plots. Both quantitative and qualitative criteria are considered to select relevant candidates for being best parameters. Our method, primarily ran on arbitrarily chosen reference forest plots, is then tested *a posteriori* on other forest plots, and proves to reproduce with a fine level of precision the complex ecological dynamics of the forest regrowth. Once calibrated, our new mathematical model can be used to simulate relevant logging scenarios, so as to better understand the temporal dynamics of forest regrowth.

*Key words.* Model Checking – Forest ecosystem – Variability – Parameter synthesis – Land-use data

# Related work

## 2.1 Paracou research station and forest data production protocol

Paracou is a research station located in Sinnamary (French Guiana), dedicated to studying the functioning of the Amazon forest ecosystem (see Figure 1). With 40 years of hindsight on forest dynamics after logging, this facility makes it possible to test the response of tropical forest ecosystems to disturbances exacerbated by global changes. Paracou station is divided into 16 forest plots, which cover a total of 125 hectares on which each of 70 000 trees has been mapped and measured at regular intervals since 1984 (see Figure 2).


**Figure 1:** Paracou research station, located in Sinnamary (French Guiana, South America). The station is divided into 16 forest plots, which cover a total of 125 hectares. In this station, 9 of the 16 forest plots have been subject to partial logging in 1988, whereas other plots serve as *control* plots.

# Related work

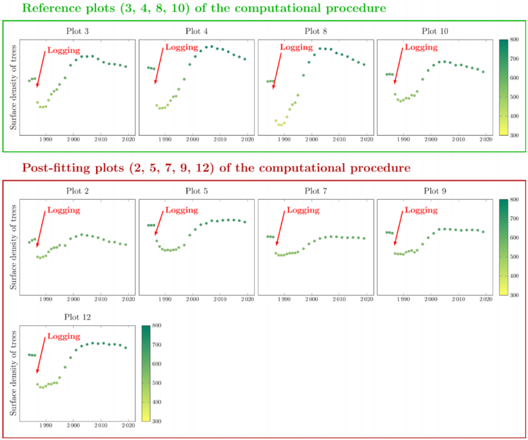
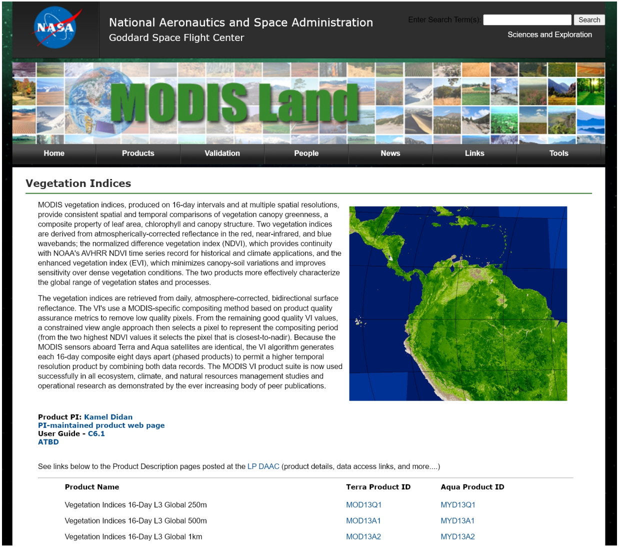



**Figure 2:** Forest data from Paracou research station. This station is divided into 16 forest plots, on which each of 70 000 trees has been mapped and measured at regular intervals since 1984. 9 of the 16 forest plots (plots 2, 3, 4, 5, 7, 8, 9, 10, 12) have been subject to partial logging in 1988, whereas other plots serve as *control* plots. Plots 3, 4, 8, 10, for which the intensity of logging is the highest, will be considered as *reference* plots for our computational procedure, whereas plots 2, 5, 7, 9, 12 will be treated *a posteriori*.

# Data base



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## Vegetation Indices

MODIS vegetation indices, produced on 16-day intervals and at multiple spatial resolutions, provide consistent spatial and temporal comparisons of vegetation canopy greenness, a composite property of leaf area, chlorophyll and canopy structure. Two vegetation indices are derived from atmospherically-corrected reflectance in the red, near-infrared, and blue wavebands; the normalized difference vegetation index (NDVI), which provides continuity with NOAA's AVHRR NDVI time series record for historical and climate applications, and the enhanced vegetation index (EVI), which minimizes canopy-soil variations and improves sensitivity over dense vegetation conditions. The two products more effectively characterize the global range of vegetation states and processes.

The vegetation indices are retrieved from daily, atmosphere-corrected, bidirectional surface reflectance. The VI's use a MODIS-specific compositing method based on product quality assurance metrics to remove low quality pixels. From the remaining good quality VI values, a constrained view angle approach then selects a pixel to represent the compositing period (from the two highest NDVI values it selects the pixel that is closest-to-nadir). Because the MODIS sensors aboard Terra and Aqua satellites are identical, the VI algorithm generates each 16-day composite eight days apart (phased products) to permit a higher temporal resolution product by combining both data records. The MODIS VI product suite is now used successfully in all ecosystem, climate, and natural resources management studies and operational research as demonstrated by the ever increasing body of peer publications.

**Product PI: Kamel Didan**
**PI-maintained product web page**
**User Guide - C6.1**
**ATBD**

See links below to the Product Description pages posted at the LP DAAC (product details, data access links, and more....)

| Product Name | Terra Product ID | Aqua Product ID |
|---|---|---|
| Vegetation Indices 16-Day L3 Global 250m | MOD13Q1 | MYD13Q1 |
| Vegetation Indices 16-Day L3 Global 500m | MOD13A1 | MYD13A1 |
| Vegetation Indices 16-Day L3 Global 1km | MOD13A2 | MYD13A2 |

# Data base

## Dataset for "Changes in Global Terrestrial Live Biomass over the 21st Century"

Xu, Liang[1]; Saatchi, Sassan S.[1]; Yang, Yan[1]; Yu, Yifan[1]; Pongratz, Julia[2]; Bloom, A. Anthony[1]; Bowman, Kevin[1]; Worden, John[1]; Liu, Junjie[1]; Yin, Yi[1]; Domke, Grant[3]; McRoberts, Ronald E.[4]; Woodall, Christopher[5]; Nabuurs, Gert-Jan[6]; de-Miguel, Sergio[7]; Keller, Michael[1]; Harris Nancy[8]; Maxwell, Sean[9]; Schimel, David[1]

Show affiliations

Live woody vegetation is the largest reservoir of biomass carbon with its restoration considered one of the most effective natural climate solutions. However, carbon fluxes associated with terrestrial ecosystems still remain the largest source of uncertainty of the global carbon balance. Here, we develop spatially explicit estimates of global carbon stock changes of live woody biomass from 2000 to 2019 using measurements from ground, air, and space. We show live biomass has removed 4.9-5.5 PgC $yr^{-1}$ from the atmosphere in this century, offsetting 4.6±0.1 PgC $yr^{-1}$ of gross emissions from land-use and environmental disturbances and adding substantially (0.23-0.88 PgC $yr^{-1}$) to the global carbon stocks. Gross emissions and removals in the tropics were four times larger than temperate and boreal ecosystems combined. Although live biomass is responsible for more than 80% of gross terrestrial fluxes, soil, dead organic matter, and lateral transport may play important roles in terrestrial carbon sink.

### Files

dataset_readme.pdf

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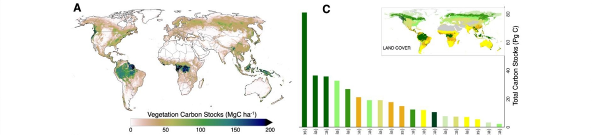

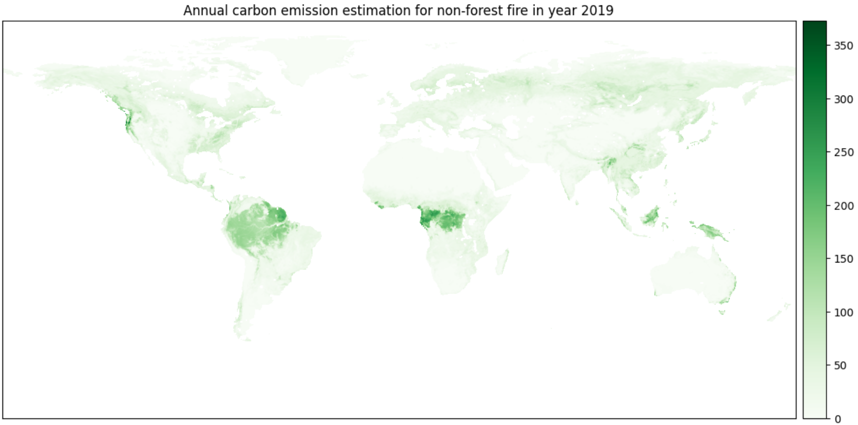
Annual carbon emission estimation for non-forest fire in year 2019
350
300
250
200
150
100
50
0

Thanks so much for your attention !